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Clock Transport Synchronization and the Dragging of Inertial Frames for Elliptical Orbits

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It is shown that is is possible to test for the dragging of inertial frames in Einstein's theory of general relativity by using the discrepancy between clocks synchronized by clock transport in elliptical orbits. Possible experiments are discussed.

A fundamental procedure in the special theory of relativity is the synchronization of standard clocks at rest in any inertial systems using light signals (Einstein, 1905; Stachel, 1980). The notation can be extended to clocks lying along a curve in a noninertial frame. Then, in general, the synchronization between the clocks at the end points of a curve depends on the particular curve chosen, *i.e.*, the synchronization is path dependent (Landau and Lifshitz, 1951; Cohen and Moses, 1977).

Clock transport is an alternative synchronization method whereby a standard clock is synchronized with a master clock and then is slowly transported to a new location to set other clocks (at rest in an inertial frame) to be at the same time. In an inertial frame, this procedure is equivalent to electromagnetic synchronization (Cohen and Moses, 1977).

In a noninertial frame, clock transport synchronization (like electromagnetic synchronization) gives a path dependent result; however, the two methods do not necessarily give the same result.

The difference between the coordinate and proper time intervals between two events A and B along a test particle trajectory

$$\left(\frac{v}{c} \ll 1, \frac{\Phi}{c^2} = -\frac{GM}{c^2 r} \ll 1\right)$$

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is (Landau and Lifshitz, 1951; Moller 1952; Adler, Bazin, and Schiffer, 1975; Arzelies, 1966; Robertson and Noonan, 1968; Rindler, 1969; Fock, 1964)

$$\Delta(t-\tau) = c^{-2} \int_{t_a}^{t_b} (\tfrac{1}{2}v^2 - \Phi)\, dt \tag{1}$$

This expression is valid for any globally time-orthogonal metric $g_{\mu\nu}$, with $g_{00} \equiv 1 + (2\Phi/c^2)$, where Φ can be interpreted as a scalar gravitational potential. The purpose of this paper is to extend the method of clock transport synchronization to stationary metrics to test for the effect of dragging of inertial frames. To do this we begin from the Brill and Cohen solution (Brill and Cohen, 1966). They have shown that, in the equatorial plane of a slowly rotating massive object, the metric can be written

$$-d\tau^2 = -A^2\, dt^2 + B^2\, dr^2 + r^2(d\Phi - \Omega\, dt)^2 \tag{2}$$

where $A^2 = B^{-2} = 1 - (2m/r)$ and $\Omega = (2J/r^3)$ with J the body's angular momentum, $J = K_+ M_+ R^2 \omega$, and $c = 1$. The generalization of expression (1), including dragging of inertial frames, is

$$\Delta\tau = \int_{t_a}^{t_b} \left(1 - \frac{v^2}{2} + \Phi + 2\dot{\Phi} K M_\oplus R_\oplus^2 \omega\right) dt \tag{3}$$

with

$$\dot{\Phi} = \frac{d\Phi}{dt}$$

For one revolution about a geosynchronous orbit (radius r, orbital angular velocity ω of the earth) we obtain (Rosenblum, 1987)

$$\Delta(t-\tau) = 3\pi\omega^2 r^2 - \frac{4\pi KMR^2\omega}{r} \tag{4}$$

The synchronization gap $\Delta(t-\tau)$ is relative to a distant inertial frame and is due to the motion of the clock and not to the choice of a noninertial frame. This synchronization gap is a cumulative effect. The first term of expression (4) corresponds to what was obtained in previous work (Cohen, Moses, and Rosenblum, 1983; Rosenblum, 1987) and for one orbit corresponds to an effect of 13.6 μsec. We are of course interested in the second term in expression (4) corresponding to the dragging of inertial frames.

An interesting question to be resolved is whether or not the use of elliptical orbits with high eccentricity will enhance this effect. After assuming an elliptical orbit in expression (3) and some algebraic manipulation, we obtain

$$\Delta(t-\tau)=\frac{3\pi MM_\oplus}{l}a(1-e^2)-\frac{4\pi KM_\oplus^u R_\oplus\omega}{a(1-e^2)} \tag{5}$$

with a the semimajor axis of the ellipse, l the angular momentum of the satellite, and e the eccentricity of the ellipse. For one revolution about a geosynchronous orbit expression (4) is obtained. The real question is whether or not, by using the $(1-e^2)^{-1}$ in expression (5), it is possible to obtain a significantly enhanced result. If we make the reasonable assumption of a perihelion distance of 6,500 km and the semimajor axis of the ellipse to be a geosynchronous orbit of approximately 42,000 km, we obtain

$$(1-\varepsilon^2)^{-1}\cong 3\times 10^1 \tag{6}$$

This, of course, gives us another order of magnitude above our previously obtained result of 1.92×10^{-17} seconds for a geosynchronous orbit (Rosenblum, 1987).

Of course, another order of magnitude helps us with possible clock accuracies of 1 part in 10^{18} and stability over a two-year period most likely occurring in the next five years (Dehmelt, 1985). Also, this additional order helps in the comparison of the clock on the satellite with the millisecond pulsar.

To summarize, the arc of an elliptical orbit leads roughly to an order of magnitude improvement in the use of clocks on satellites to detect the dragging of inertial frames.

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